MODE SWITCHING IN A SUPERSONIC BOUNDARY LAYER

A. V. Fedorov and A. P. Khokhlov UDC 532.526

The initial stage of the laminar–turbulent transition in a boundary layer is characterized by the development of unstable modes [1, 2]. If the level of external disturbances is low, the extent of this stage is commensurate with the space scale of nonparallelism effects in the average flow. The downstream buildup of the boundary layer tends to distort the characteristics of the individual modes, on the one hand, and to induce mode switching, on the other.

The only unstable mode at subsonic and moderate supersonic flow velocities is the first mode (the Tollmien–Schlichting wave), whose parameters differ significantly from the parameters of other normal modes. Mode switching is weak in this case, and the development of the Tollmien–Schlichting wave can be treated separately. Numerous calculations of the evolution of unstable disturbances in a boundary layer have been carried out in this setting [1, 2]. They provide the basis of the e^N-method for computing the Reynolds numbers at which laminar–turbulent transition begins [3].

At sufficiently high Mach numbers (the specific values of which may be found in [4]) qualitative changes take place in the spectrum of normal modes, where the second, third, and higher modes acquire an acoustic instability. Acoustic instability is attributable to mode locking in the discrete spectrum [5, 6]. In the vicinity of mode locking the dispersion curves $\alpha(\omega)$ (α is the wave number, and ω is the frequency of the disturbance) split, one mode acquiring a positive increment of its growth rate, and the other acquiring a negative increment. If the split is sufficiently wide, a zone of instability is formed with the branch points of the spectrum situated near its boundaries [5-7]. The eigenvalues of two modes merge in the vicinity of the branch points, so that strong mode switching is possible here as a result of nonparallelism of the average flow. Similar anomalies occur in a thin shock layer [6, 8, 9].

In the present article we analyze mode switching near the branch points of the spectrum in the example of a supersonic boundary layer. The results are quite general and can be applied to other classes of unstable, slightly nonparallel flows.

1. We consider two-dimensional flow in a laminar boundary layer having a characteristic thickness δ. We assume that the parameters of the main flow change downstream by an order of magnitude within the scale $L \gg \delta$. We reduce the longitudinal (x_1), lateral (y), and transverse (z) coordinates to dimensionless form relative to δ, and likewise the time t relative to δ/U_e (U_e is a characteristic velocity at the outer boundary of the boundary layer). We represent a disturbance with a fixed frequency $\omega \neq 0$ and a fixed z-component of the wave vector β by the expression $\mathbf{Q} = \mathbf{F}(x_1, y)\exp(i\beta z - i\omega t)$.

We introduce the "slow" variable $x = \varepsilon x_1$ (the small parameter $\varepsilon = \delta/L$ characterizes the nonparallelism of the main flow). The amplitude $\mathbf{F}$ obeys the linearized Navier–Stokes equations, which can be written in the operator form

$$H(y, \partial_y, x, \varepsilon\partial_x, \beta, \omega)\mathrm{F} = 0; \ |\mathrm{F}| \to 0, \ y \to \infty; \tag{1.1}$$

$$N(\partial_y, x, \beta, \omega)\mathrm{F}(x, 0) = 0, \tag{1.2}$$

where Eq. (1.2) gives uniform boundary conditions on the surface of the body y = 0; the matrices H and N depend on the parameters of the main flow, and their explicit form may be found in [1, 10].

We are interested in the particular solution of the problem (1.1), (1.2) in the form of a superposition of discrete spectrum modes propagating downstream from a certain source of excitation:

Zhukovskii. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 6, pp. 67-72, November-December, 1992. Original article submitted May 16, 1991; revision submitted September 20, 1991.

0021-8944/92/3306-0831$12.50 © 1993 Plenum Publishing Corporation

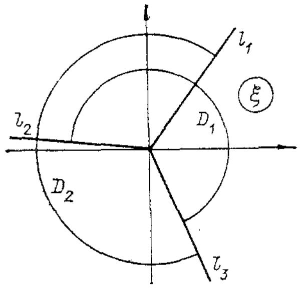

Fig. 1

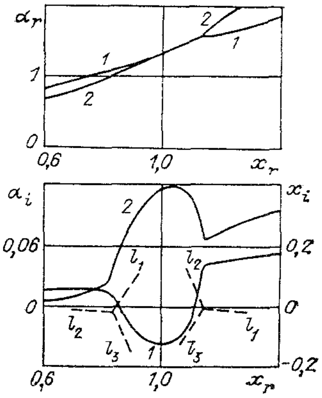

Fig. 2

$$\mathbf{F}=\sum_k(\mathbf{F}_{k0}+\varepsilon\mathbf{F}_{k1}+\ldots)\exp\left(i\varepsilon^{-1}S_k\right),\quad S_k=\int\limits_{x_S}^{x}\alpha_k(x)\,dx. \tag{1.3}$$

We assume that the mode amplitudes are give in the initial cross section x_S. In the principal approximation with respect to ε we obtain a locally uniform problem containing x as a parameter:

$$H(y,\ \partial_y,\ x,\ i\alpha,\ \beta,\ \omega)\mathbf{F}_0=0,\ N\mathbf{F}_0(x,\ 0)=0,\ |\mathbf{F}_0|\to 0,\ y\to\infty. \tag{1.4}$$

We drop the subscript k from now on. The solution $\mathbf{F}_0$ is conveniently written in the form of the product of the eigenfunction of the problem (1.4) $\mathbf{A}(x, y, \alpha)$ and a certain predetermined normalization to a function C(x). The equation for the next-higher approximation $\mathbf{F}_1$ is solvable if its right-hand side is orthogonal to the eigenfunction $\mathbf{B}$ of the conjugate problem, i.e., if

$$\begin{gathered}\mathbf{F}_0=C(x)\mathbf{A}(x,\ y,\ \alpha),\\ \left\langle\mathbf{B},\frac{\partial H}{\partial\alpha}\mathbf{A}\right\rangle\frac{dC}{dx}+\left\langle\mathbf{B},\frac{\partial H}{\partial\alpha}\frac{\partial\mathbf{A}}{\partial x}+\frac{1}{2}\frac{d\alpha}{dx}\frac{\partial^2H}{\partial\alpha^2}\mathbf{A}\right\rangle C=0,\\ C(x_S)=C_S,\quad\langle\mathbf{f},\mathbf{g}\rangle=\int\limits_0^\infty\sum_j f_j^*g_j\,dy\end{gathered} \tag{1.5}$$

(the asterisk denotes complex conjugate). The system (1.4), (1.5) describes the evolution of a mode of the boundary layer without mode switching. The local behavior of the eigenvalue $\alpha(x)$ can be determined by replacing $\mathbf{F}_0$ with $\mathbf{A}$ in Eq. (1.4), differentiating the system of equations with respect to x, and writing the solvability condition for the resulting problem [10] in the form

$$\left\langle\mathbf{B},\frac{\partial H}{\partial\alpha}\mathbf{A}\right\rangle\frac{d\alpha}{dx}+\left\langle\mathbf{B},\frac{\partial H}{\partial x}\mathbf{A}\right\rangle=0. \tag{1.6}$$

The expansion (1.3) is applicable in domains where none of the eigenvalues is equal to zero and each one is distinct from all others [11]. For disturbances with a nonzaro frequency the first condition is equivalent to the nonexistence of modes having an infinite phase velocity and is valid for real physical problems; the second condition implies that the spectrum does not contain any branch points. We now determine the cases in which this condition fails. Let the eigenvalue α_0 and the eigenfunction $\mathbf{A}_0=\mathbf{A}(x_0,\ y,\ \alpha_0)$ exist at the point x_0, which is complex in general. In a small neighborhood of x_0 we write the eigenfunction in the series form

$$\mathbf{A}=\mathbf{A}_0(y)+\varkappa\mathbf{A}_1(y)+\varkappa^2\mathbf{A}_2(y)+\ldots,\ \varkappa=\alpha-\alpha_0. \tag{1.7}$$

Assuming that the operator H is analytic in the neighborhood of the point (x_0, α_0), we expand it into a series, substitute Eq. (1.7) into (1.4), and form the scalar product with the solution of the conjugate problem $\mathbf{B}_0=\mathbf{B}(x_0,\ y,\ \alpha_0)$. As a result, we obtain an equation relating α and x:

$$\left\langle \mathbf{B}_0, \frac{\partial H_0}{\partial \alpha}\mathbf{A}_0\right\rangle \varkappa + \left\langle \mathbf{B}_0, \frac{\partial H_0}{\partial x}\mathbf{A}_0\right\rangle X + \left\langle \mathbf{B}_0, \frac{\partial H_0}{\partial \alpha}\mathbf{A}_1 + \frac{1}{2}\frac{\partial^2 H_0}{\partial \alpha^2}\mathbf{A}_0\right\rangle \varkappa^2 + \ldots = 0, \quad X = x - x_0. \tag{1.8}$$

The point x_0 is a lowest-order branch point if

$$\left\langle \mathbf{B}_0, \frac{\partial H_0}{\partial \alpha}\mathbf{A}_0\right\rangle = 0, \quad \left\langle \mathbf{B}_0, \frac{\partial H_0}{\partial \alpha}\mathbf{A}_1 + \frac{1}{2}\frac{\partial^2 H_0}{\partial \alpha^2}\mathbf{A}_0\right\rangle \neq 0; \tag{1.9}$$

$$\alpha = \alpha_0 \pm \lambda\sqrt{X} + \ldots, \quad \lambda^2 = -\left\langle \mathbf{B}_0, \frac{\partial H_0}{\partial x}\mathbf{A}_0\right\rangle \Big/ \left\langle \mathbf{B}_0, \frac{\partial H_0}{\partial \alpha}\mathbf{A}_1 + \frac{1}{2}\frac{\partial^2 H_0}{\partial \alpha^2}\mathbf{A}_0\right\rangle. \tag{1.10}$$

This is the most typical situation, because higher-order branching requires the application of additional orthogonality conditions. We now verify that the expansion (1.7) is sensible. To do so, we substitute it into Eq. (1.4) and set the sums of coefficients of like powers of X equal to zero, taking Eq, (1.10) into account:

$$\begin{gathered} H_0\mathbf{A}_0 = 0, \quad H_0\mathbf{A}_1 + \frac{\partial H_0}{\partial \alpha}\mathbf{A}_0 = 0, \\ H_0\mathbf{A}_2 + \lambda^{-2}\frac{\partial H_0}{\partial x}\mathbf{A}_0 + \frac{1}{2}\frac{\partial^2 H_0}{\partial \alpha^2}\mathbf{A}_0 + \frac{\partial H_0}{\partial \alpha}\mathbf{A}_1 = 0. \end{gathered} \tag{1.11}$$

By virtue of Eqs. (1.9) and (1.10) the right-hand sides of the second and third equations are orthogonal to be conjugate function $\mathbf{B}_0$, so that the solutions $\mathbf{A}_1, \mathbf{A}_2\ldots$ exist.

To determine the asymptotic behavior of C(X) in the limit $X \to 0$, we substitute Eq. (1.7) and the analogous expansion for the conjugate problem into Eqs. (1.5) and (1.6), taking Eq. (1.10) into account, whereupon we obtain

$$C(X) = C_0X^{-1/4} + \ldots, \; X \to 0. \tag{1.12}$$

The principal term of the expansion has a universal form, i.e., only the constant C_0 and subsequent regular terms of the asymptotic expansion change under a change of normalization of the eigenfunction $\mathbf{A}$. The singular behavior of the function C(X) in the vicinity of a branch point is determined entirely by the behavior of the spectrum $\alpha(x)$. Taking Eqs. (1.10) and (1.12) into account, we write the principal term of the expansion (1.3) in the limit $X \to 0$ in the form

$$\begin{gathered} \mathbf{F} \sim [a\exp(\varphi) + b\exp(-\varphi)]X^{-1/4}\mathbf{A}_0\exp[i\varepsilon^{-1}(S_0 + \alpha_0X)], \\ \varphi = (2/3)i\varepsilon^{-1}\lambda X^{3/2}. \end{gathered} \tag{1.13}$$

If $X = O(\varepsilon^{2/3})$, the eikonal representation of (1.3) is invalid. In the interior domain with the variable $\xi = \varepsilon^{-2/3}X$ we have

$$\mathbf{F} = \varepsilon^{-1/6}\exp(i\varepsilon^{-1}S_0 + i\varepsilon^{-1/3}\alpha_0\xi)[f_0(\xi)\mathbf{A}_0 + \varepsilon^{1/3}f_1(\xi)\mathbf{A}_1 + \varepsilon^{2/3}f_2(\xi)\mathbf{A}_2 + \ldots] + \ldots; \tag{1.14}$$

$$H_0\mathbf{A}_0 = 0, \quad f_1H_0\mathbf{A}_1 - if_0'\frac{\partial H_0}{\partial \alpha}\mathbf{A}_0 = 0; \tag{1.15}$$

$$f_2H_0\mathbf{A}_2 + \xi f_0\frac{\partial H_0}{\partial x}\mathbf{A}_0 - if_1'\frac{\partial H_0}{\partial \alpha}\mathbf{A}_1 - \frac{1}{2}f_0''\frac{\partial^2 H_0}{\partial \alpha^2}\mathbf{A}_0 = 0 \tag{1.16}$$

(the prime signifies differentiation with respect to ξ). Making use of Eq. (1.11), from the second Eq. (1.15) we obtain $f_1 = -if_0'$. It follows from the solvability condition (1.16) and the relation (1.10) that

$$f_0'' + \lambda^2 \xi f_0 = 0. \tag{1.17}$$

The asymptotic behavior of the solutions of the Airy equation (1.17) depends on the orientation of the Stokes lines ℓ_j, $j = 1, 2, 3$, which are determined from the condition $\mathrm{Re}\Delta(\xi) = 0$, $\Delta(\xi) = (2/3)i\lambda\xi^{3/2}$.

Let the Stokes lines run through the complex plane of ξ as shown in Fig. 1. Following Fedoryuk [12], we introduce matched canonical domains $D_j : \ell_j \in D_j$, $\ell_{k\neq j} \in \partial D_j$. We choose a branch of the function $\Delta(\xi)$ in each domain in such a way that $\mathrm{Im}\Delta > 0$, $\xi \in \ell_j \in D_j$. Then $\mathrm{Re}\Delta > 0(<0)$ to the right (to the left) of ℓ_j, and the solution (1.17) has the asymptotic representation

$$\begin{gathered} f_0 \sim c_j \xi^{-1/4}[a_j \exp(\Delta) + b_j \exp(-\Delta)], \ |\xi| \to \infty, \\ \xi \in D_j, \ |c_j| = 1, \ \arg(c_j \xi^{-1/4}) = 0, \ \xi \in l_j, \ j = 1, 2, 3; \end{gathered} \tag{1.18}$$

$$\begin{bmatrix} a_{j+1} \\ b_{j+1} \end{bmatrix} = \exp(-i\pi/6) \begin{bmatrix} 0 & 1 \\ 1 & i \end{bmatrix} \begin{bmatrix} a_j \\ b_j \end{bmatrix}, \ j = 1, 2. \tag{1.19}$$

We continue the Stokes lines into the domain $X = o(1)$, where the asymptotic expansion (1.13) is valid. We chose the branches of the function $\varphi(X)$ in the same way as for $\Delta(\xi)$, set $(a, b) = c_j(a_j, b_j)$, and require that $\arg(c_j X^{-1/4}) = 0$, $X \in \ell_j$; then the asymptotic expansions (1.13) and (1.18) match. Equation (1.19) describes mode switching when the real x axis runs from one canonical domain into another, intersecting one of the Stokes lines. The transition matrix has a universal form, applying to any type of slightly nonuniform flow whose spectrum contains a branch point of the type in question. It follows from Eq. (1.19) that the mode amplitudes change by their principal order, so that mode switching effects cannot be ignored.

2. A similar result can be obtained by expanding the amplitude of the disturbance $\mathbf{F}$ into a biorthogonal system of eigenfunctions of the locally uniform problem and using the formalism of [1] in the two-mode approximation. We write the linearized Navier–Stokes equations for the amplitude of a disturbance with a fixed frequency and the particular solution of these equations in the form of a two-mode sum:

$$\begin{gathered} (H_0(y, \partial_y, x, \beta, \omega) + \varepsilon H_1(y, \partial_y, x, \beta, \omega)\partial x)\mathbf{F} = 0, \ N\mathbf{F}(x, 0) = 0, \\ \mathbf{F} = \sum_{j=1}^{2} c_j(x)\, \mathbf{A}_j(x, y) \exp(i\varepsilon^{-1} S_j). \end{gathered} \tag{2.1}$$

The eigenfunctions $\mathbf{A}_j$ and the coefficients c_j are the solutions of the equations

$$\begin{gathered} (H_0 + i\alpha_j H_1)\mathbf{A}_j = 0, \ N\mathbf{A}_j(x, 0) = 0, \ |\mathbf{A}_j| \to 0, \ y \to \infty; \\ \frac{dc_j}{dx} = \sum_{k=1}^{2} W_{jk} c_k \exp[i\varepsilon^{-1}(S_k - S_j)], \quad j = 1, 2; \end{gathered} \tag{2.2}$$

$$W_{jk} = -\frac{\left\langle \mathbf{B}_j, H_1 \dfrac{\partial \mathbf{A}_k}{\partial x} \right\rangle}{\langle \mathbf{B}_j, H_1 \mathbf{A}_j \rangle}. \tag{2.3}$$

Expanding $\mathbf{A}_j$, $\alpha_j(x)$, and the operators H_0 and H_1 in the vicinity of the branch point x_0 and invoking the solvability conditions for the first and second approximations, in the limit $X \to 0$ we have

$$\begin{gathered} \alpha_{1,2} = \alpha_0 \pm \lambda\sqrt{X} + \ldots, \ \mathbf{A}_{1,2} = \mathbf{A}^0 + (\alpha_{1,2} - \alpha_0)\mathbf{A}^1 + \ldots, \\ H_1 = H_{10} + XH_{11} + \ldots, \ H_0 = H_{00} + XH_{01} + \ldots, \\ \langle \mathbf{B}^0, H_{10}\mathbf{A}^0 \rangle = 0, \ \lambda = \sqrt{-i\langle \mathbf{B}^0, H_{01}\mathbf{A}^0 \rangle / \langle \mathbf{B}^0, H_{10}\mathbf{A}^0 \rangle}. \end{gathered}$$

Analogous expansions can be made for the solution $\mathbf{B}_j$ of the conjugate problem. Substituting them into Eq. (2.3), we find that the matrix elements have the universal form $W_{jk} = (-1)^{f+k-1}/4X$ in the vicinity of the branch point. Solving the system (2.2), we readily show that the asymptotic expansion of F in the limit $X \to 0$ is analogous to Eq. (1.13). In the interior domain $\xi = O(1)$

$$c_1' = [-c_1 + c_2 \exp(-4i\lambda\xi^{3/2}/3)]/(4\xi),$$
$$c_2' = [c_1 \exp(4i\lambda\xi^{3/2}/3) - c_2]/(4\xi).$$

Denoting $\Phi_{1,2} = c_1\exp(2i\lambda\xi^{3/2}/3) \pm c_2\exp(-2i\lambda\xi^{3/2}/3)$, we arrive at an equation for Φ_1 that coincides with the equation for f_0 (1.17). Consequently, both approaches give the same result for mode switching.

3. As an example, we consider the boundary layer on a plate in a supersonic flow of an ideal gas. We adopt the displacement thickness $\delta^* \sim \sqrt{\nu_e x^*/U_e}$ as the characteristic length scale. We define "fast" and "slow" longitudinal coordinates by the equations $x_1 \equiv R = \delta^* U_e/\nu_e$ and $x = \varepsilon R$. We make the parameter ε equal to the reciprocal of the Reynolds number corresponding to the maximum growth rate of the unstable mode; then $x = 1$ approximately at the center of the instability zone. The eigenvalue problem (1.4) reduces to a Lees–Lin system (see, e.g. [1, 4]). The conditions at the wall stipulate that the velocity and temperature perturbations are equal to zero there.

By virtue of self-similarity the profiles of the average flow depend only on the variable y to within $O(\varepsilon)$, so that the Reynolds number enters into the equations explicitly. This fact simplifies considerably the calculation of the eigenvalues α for complex-valued R, which is necessary in looking for branch points and plotting the Stokes lines.

Figure 2 shows the real parts α_r and the imaginary parts α_i of the wave numbers as functions of the coordinate x for the first and second modes (curves 1 and 2). The calculations are carried out for the following parameters: Mach number 6; Prandtl number 0.72; adiabatic exponent 1.4; temperature factor $T_w/T_{av} = 0.2$; dimensionless frequency parameter $f = \omega^*\nu_e/U_e^2 = 2.7722 \cdot 10^{-4}$. Using a power-law dependence of the viscosity coefficient on the temperature with power exponent $n = 0.75$, we obtain the parameter $\varepsilon = 2.276 \cdot 10^{-4}$. The eigenvalues split in the interval $0.87 \lesssim x \lesssim 1.15$, imparting an instability to the first mode. The branch points x_0 are sought by the condition (1.9), and their values are $x_0^{(1)} \approx 0.853 - i2.21 \cdot 10^{-2}$ and $x_0^{(2)} \approx 1.143 - i5.04 \cdot 10^{-3}$. The Stokes lines are represented by the dashed lines ℓ_1 emanating from the point $x_0^{(1)}$ and ℓ_2 emanating from $x_0^{(2)}$, and they intersect the real axis at the points $x_*^{(1)} \approx 0.869$ and $x_*^{(2)} \approx 1.140$, respectively.

This information is sufficient for describing the evolution of modes with allowance for mode switching. Suppose, for example, that only a stable second mode is generated in the upstream domain $x < x_*^{(1)}$, i.e., the amplitude coefficients in the expansion (1.3) have the values $a = h$ and $b = 0$. Using the transition matrix (1.19), we find that $|a| = |b| = |h|$ for $x > x_*^{(1)}$. Consequently, as the stable second mode passes through the neighborhood of the branch point $x_0^{(1)}$, it excites an unstable first mode of equal amplitude. The case in which only the first mode is present in the upstream domain is treated analogously. This mode passes through the branch point $x_0^{(1)}$, changing phase, and does not excite (in the principal approximation with respect to ε) the second mode.

In closing, we note that the foregoing results can be used to extend the e^N-method to classes of flows whose normal modes have a branching spectrum with several instability zones. This situation is typical of inviscid disturbances in hypersonic boundary layers [4-7] and in a thin shock layer [6, 8, 9]. In calculating the total magnification exponent N, it is necessary to switch from one unstable mode to another by the rules (1.19), having first computed the branch points and plotted the Stokes lines.

LITERATURE CITED

1. V. N. Zhigulev and A. M. Tumin, Onset of Turbulence [in Russian], Nauka, Novosibirsk (1987).
2. S. A. Gaponov and A. A. Maslov, Evolution of Disturbances in Compressible Flows [in Russian], Nauka, Novosibirsk (1980).
3. J. N. Hefner and D. M. Bushnell, "Application of stability theory to laminar flow control," AIAA Paper No. 1493, AIAA, New York (1979).

4. L. M. Mack, "Boundary-layer stability theory, Part 2," Jet Propulsion Lab. Doc, No. 900-277 (Rev. A), Pasadena, CA (1969).
5. V. R. Gushchin and A. V. Fedorov, "Excitation and development of unstable disturbances in a supersonic boundary layer," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 3 (1990).
6. V. R. Gushchin and A. V. Fedorov, "Qualitative characteristics of the instability of flows near a wall at large flow velocities," in: Models of the Mechanics of Inhomogeneous Systems [in Russian], ITPM SO AN SSSR, Novosibirsk (1989).
7. G. V. Petrov, "Two-dimensional absolute instability of a supersonic boundary layer," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1 (1988).
8. G. V. Petrov, "Stability of thin shock layer on wedge in hypersonic flow of a perfect gas," in: Laminar—Turbulent Transition: IUTAM Symposium, Novosibirsk (1984).
9. V. R. Gushchin and A. V. Fedorov, "Short-wave instability in a shock layer of an ideal gas," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1 (1989).
10. A. H. Nayfeh, "Stability of three-dimensional boundary layers," AIAA J., 18, No. 4 (1980).
11. S. A. Lomov, Introduction to the General Theory of Singular Perturbations [in Russian], Nauka, Moscow (1981).
12. M. V. Fedoryuk, Asymptotic Methods for Linear Ordinary Differential Equations [in Russian], Nauka, Moscow (1983).

TURBULENT FLOW OF A GAS SUSPENSATE WHOSE PARTICLES INTERACT STRONGLY WITH THE CHANNEL WALLS

I. V. Derevich

UDC 532.529

Turbulent two-phase gas flows are widely used in power engineering, aviation, and chemical engineering. In the pneumatic transport of a powder, one frequently has fairly coarse particles, whose dynamic relaxation times may greatly exceed the characteristic time scale of the turbulent pulsations. In that case, the pulsating and average motion of the powder is substantially different from that for small particles, whose dynamic relaxation times are less than or comparable with the time scale of the velocity pulsations in the liquid phase. The extent of the pulsating motion for small particles is determined by the extent to which the powder is extrained in the turbulent motion and can be estimated in the local-equilibrium approximation without considering the collisions of the particles with the channel walls [1]. The average and pulsating characteristics for large particles are dependent on the interaction with the walls. There are effects from the marked velocity difference between the phases and the intense chaotic motion of the powder, where the level of the pulsating motion for the powder may greatly exceed that of the particle pulsation in an unbounded space with identical turbulence intensity, and this can be explained only on the basis of the collisions between inertial particles and the bounding surfaces. The collisions cause the particles to lose momentum and to rotate around the points of contact. The Magnus force arising from the rotation causes rapid transverse displacement [2, 3]. The channel walls in a gas—power system thus provide positive feedback, which causes additional pulsations in the powder by comparison with turbulent flow in an unbounded space.

There are two approaches to calculating the characteristics of such flows. Firstly, there is direct stochastic simulation, which is based on solving the equations of motion for a single particle in a random velocity pattern [2, 5-8]. However, to obtain information on the averaged characteristics, it is necessary to calculate many thousands of such paths, which consumes considerable time. In spite of the apparent simplicity, the method of calculating Lagrange paths is not widely used in designing pneumatic transport systems. The second method is based on the conservation equations for mass, momentum, and angular momentum of the particles and the intensity of the turbulent pulsations [3, 9]. Then to close the system, it is necessary to derive expressions representing the rate of turbulent momentum transport, the angular momentum, and the pulsation energy, and also to substitute boundary conditions for the equations for the first and second moments, which incorporate the inter-

Moscow. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 6, pp. 73-81, November-December, 1992. Original article submitted November 25, 1991.

0021-8944/92/3306-0836$12.50 © 1993 Plenum Publishing Corporation